Sec. 11.1 Power Functions

Power Function – has degree n and is in the form $f(x) = kx^p$, where k and p are constants.

Degree – the power that the function is taken to

Ex: What degree is the function? Is it odd or even?

a. $f(x) = 2x^3$
THIRD DEGREE
(CUBIC)
ODD
{Symmetric about origin}

b. $f(x) = -3x^8$
EIGHTH DEGREE
EVEN
{Symmetric about y-axis}

c. $f(x) = -3x^7$
SEVENTH DEGREE
ODD

d. $f(x) = 8x^2$
SECOND DEGREE
(QUADRATIC)
EVEN

NOTE: This determines which properties the function will have.

A quantity y is **(directly) proportional to a power** of x if $y = kx^p$, k and p are constants.

Ex: The area, A, of a circle is proportional to the square of its radius, r: $A = \pi r^2$.

Why? As the length of the radius increases, so does the area. $K = \pi$

A quantity y is **inversely proportional** to x^n if $y = \dfrac{k}{x^p}$, where k and p are constants.

Ex: The weight, w, of an object is inversely proportional to the square of the object's distance, d, from the earth's center: $w = \dfrac{k}{d^2} = kd^{-2}$.

Why? As the distance from the earth's center increases, the weight decreases.

A **power function** is a function of the form $f(x) = kx^p$, where k and p are constants.

Ex: Which of the following functions are power functions? For each power function, state the value of the constants k and p in the formula $y = kx^p$.

(a) $f(x) = 13\sqrt[3]{x}$

$f(x) = 13x^{\frac{1}{3}}$

$K = 13$

$p = \frac{1}{3}$

(b) $g(x) = 2(x+5)^3$

Directly proportional to $(x+5)^3$ but not directly proportional to a power of X.

(c) $u(x) = \sqrt{25/x^3}$

$u(x) = \dfrac{\sqrt{25}}{\sqrt{x^3}} = \dfrac{5}{x^{\frac{3}{2}}}$

$u(x) = 5x^{-\frac{3}{2}}$

$K = 5 \quad p = -\frac{3}{2}$

(d) $v(x) = 6 \cdot 3^x$

NOT a power function – exponential

The Effect of the Power p

Properties of Power Functions, n is an even function:

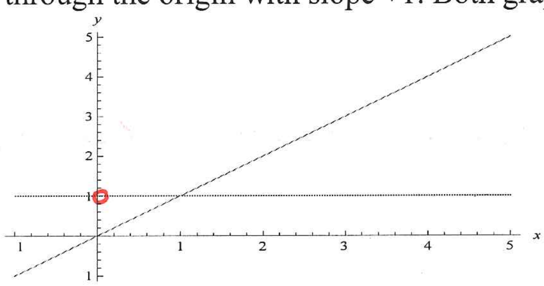
a. The graph is symmetric with respect to the y-axis if *f* is even.
b. The domain is the set of all real numbers. The range is the set of nonnegative real numbers.
c. The graph always contains the points (0, 0), (1, 1), and (-1, 1).
d. As the exponent n increases in magnitude, the graph becomes more vertical when x < -1 or x > 1; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.
e. Graph will be a parabola.

Properties of Power Functions, n is an odd function:

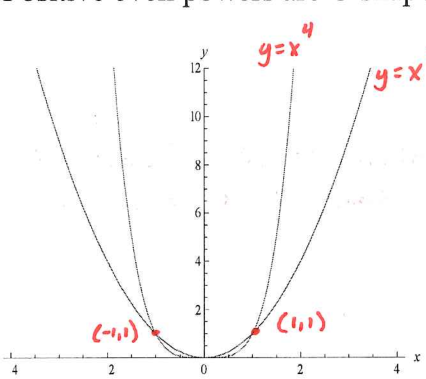
a. The graph is symmetric with respect to the origin if *f* is odd.
b. The domain and range are the set of all real numbers.
c. The graph always contains the points (0, 0), (1, 1), and (-1, -1).
d. As the exponent n increases in magnitude, the graph becomes more vertical when x < -1 or x > 1, but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.
e. Graph will look like the cube function.

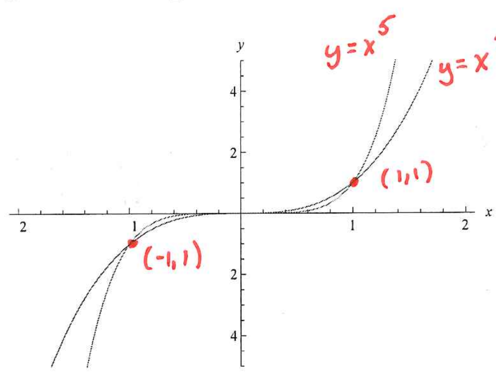
Graphs of the Special Cases $y = x^0$ and $y = x^1$

The power functions corresponding to p = 0 and p = 1 are both linear. The function $y = x^0 = 1$, except at $x = 0$. Its graph is a horizontal line with a hole at (0,1). The graph of $y = x^1 = x$ is a line through the origin with slope +1. Both graphs contain the point (1,1).



Positive even powers are U shaped and positive odd powers are chair shaped.

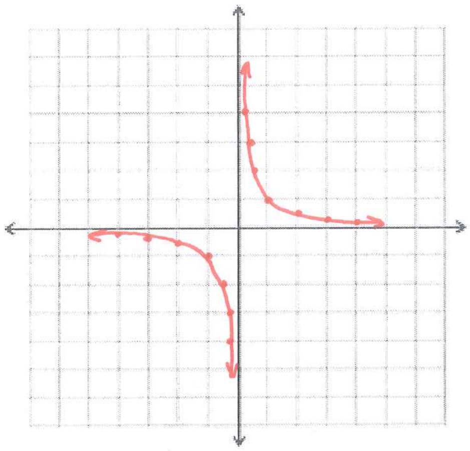


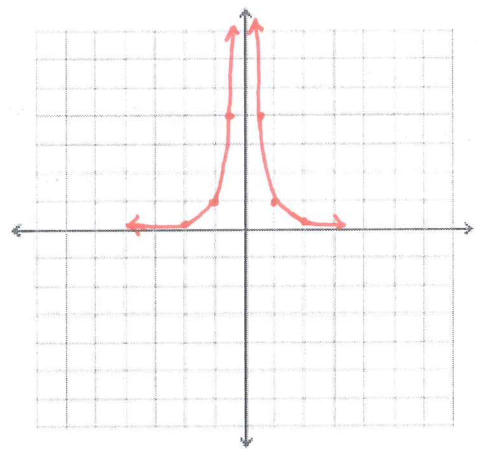


Ex: Now graph the negative odd function $f(x)=x^{-1}$ and the negative even function $f(x)=x^{-2}$. Then find the limits below.

$f(x)=\frac{1}{x}$

x	$f(x)$
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
0	$\emptyset$
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
$\frac{1}{2}$	2
$\frac{1}{3}$	3





$f(x)=\frac{1}{x^2}$

x	$f(x)$
-3	$\frac{1}{9}$
-2	$\frac{1}{4}$
-1	1
0	$\emptyset$
1	1
2	$\frac{1}{4}$
3	$\frac{1}{9}$
$\frac{1}{2}$	4
$\frac{1}{3}$	9
$-\frac{1}{3}$	9

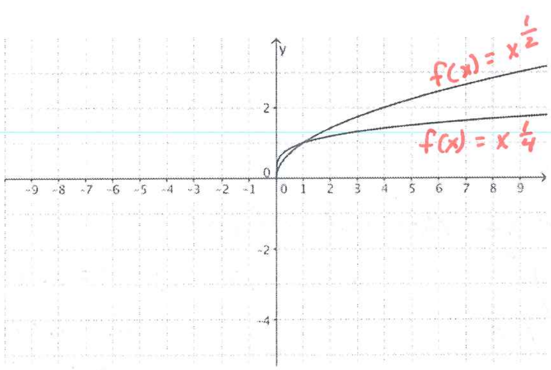
$\lim_{x\to\infty}\left(\frac{1}{x}\right)=0 \quad \lim_{x\to-\infty}\left(\frac{1}{x}\right)=0$

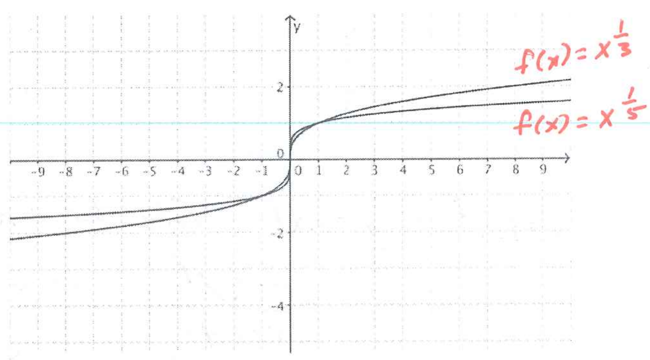
$\lim_{x\to0^+}\left(\frac{1}{x}\right)=\infty \quad \lim_{x\to0^-}\left(\frac{1}{x}\right)=-\infty$

$\lim_{x\to\infty}\left(\frac{1}{x^2}\right)=0 \quad \lim_{x\to-\infty}\left(\frac{1}{x^2}\right)=0$

$\lim_{x\to0}\left(\frac{1}{x^2}\right)=\infty$

Ex: The graphs below are for positive fractional powers. Match each graph with its correct equation: $f(x)=x^{\frac{1}{2}}$, $f(x)=x^{\frac{1}{3}}$, $f(x)=x^{\frac{1}{4}}$, and $f(x)=x^{\frac{1}{5}}$. Describe the characteristics of the graphs and the differences between the functions with the even and odd exponents.





$x^{\frac{1}{4}}$ is steeper near the origin but flatter away from it. Both graphs are increasing and concave down (slower and slower rate of increase). Since $f(x)=\sqrt{x}$ and $f(x)=\sqrt[4]{x}$, they have only non-negative values in the domain. All even fractional exponents of the form $\frac{1}{n}$ will look like this.

Same properties as the even fractional powers except there are negative values in the domain and the graphs are symmetrical about the origin.

$f(x)=\sqrt[3]{x}$ will rise faster

$f(x)=\sqrt[5]{x}$

Finding the Formula for a Power Function

Ex: From geometry, we know that the radius of a sphere is directly proportional to the cube root of the volume. In a sphere of radius 18.2 cm has a volume of 25,252.4 cubic cm, what is the radius of a sphere whose volume is 30,000 cubic cm?

$r = KV^{\frac{1}{3}}$

$18.2 = K(25{,}252.4)^{\frac{1}{3}}$

$\frac{18.2}{(25{,}252.4)^{\frac{1}{3}}} = K$

$.620 \approx K$

$r = .620V^{\frac{1}{3}}$

$r = .620(30{,}000)^{\frac{1}{3}}$

$\boxed{r = 19.26 \text{ cm}}$

Ex: The weight of a burrito in ounces, w, is inversely proportional to the square root of the hunger level, h, (measured in international hunger units) of the person ordering the burrito. A person with a hunger level of 64 IHU orders of 2.5 ounce burrito. Write an equation for the weight of a burrito and find the weight of a burrito ordered by someone with a hunger level of 200 IHU. What type of person gets the biggest burrito?

$w = \frac{K}{\sqrt{h}}$

$2.5 = \frac{K}{64^{\frac{1}{2}}}$

$2.5 = \frac{K}{8}$

$20 = K$

$w = \frac{20}{h^{\frac{1}{2}}}$

$\boxed{w = 20h^{-\frac{1}{2}}}$

$w = 20(200)^{-\frac{1}{2}}$

$\boxed{w = 1.414 \text{ oz}}$

The people who are the least hungry will get the biggest burrito since weight and hunger level are inversely proportional.

Ex: Water is leaking out of a container with a hole in the bottom. Torricelli's Law states that at any instant, the velocity v with which water escapes from the container is a power function of d, the depth of the water at that moment. When d is 9 feet, then the velocity is 24 feet/sec and when d is $\frac{1}{4}$ feet, the velocity is 4 feet/sec. Express v as a function of d.

$v = Kd^p$ or $y = kx^p$ Take the ratio of any two ordered pairs.

$\frac{v = Kd^p}{v = Kd^p}$

$\frac{24 = K \cdot 9^p}{4 = K(\frac{1}{4})^p}$

$6 = \frac{9^p}{\frac{1}{4}^p}$

$6 = \left(\frac{9}{\frac{1}{4}}\right)^p$

$6 = 36^p$ *

$\log 6 = \log(36^p)$

$\frac{\log 6}{\log 36} = \frac{p \log 36}{\log 36}$

$\frac{1}{2} = p$ *

SOLVE FOR K: $v = Kd^p$

$24 = K \cdot 9^{\frac{1}{2}}$

$24 = K \cdot 3$

$8 = K$

$\boxed{v = 8d^{\frac{1}{2}}}$

HW: pg 437-441 #S1-S10,2-4,7,9-12,14,17,18,21,22,25,26,29,32-34,38,41,43,46